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Semi-T_1 Spaces of Semi-Separation axioms in topological space

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Abstract : In this paper, we introduce a new class of space in the topological space, namely Semi-T_1 space of Semi-Separation axioms in the topological space. We find characterizations of these spaces. Further, we study some fundamental properties of these spaces in the topological space.

Keywords : Semi-open set, Semi-closed set, Semi-closure.

I. Introduction

The term Semi-T_1 space is weaker form of T_1 space. It plays a significant role in the topological space. Ever since the concept of the term Semi-T_0 space & Semi-T_1 space in a topological space was first introduced by the mathematician S. N. Maheshwari & R. Prasad[3] in the year 1975. The term Semi-open sets are introduced by N. Levine[1] in 1963. Then after the mathematician S. N. Maheshwari & R. Prasad[3] used Semi-open sets to define and introduced the Separation axioms called Semi-Separation axioms like Semi-T_0 space in the year 1975. Later, the mathematician P. Bhattacharya & B. K. Lahiri[6] used to Semi-open sets to define the axiom Semi-$T_{1/2}$ space in the year 1982 and further investigated the separation axioms like Semi-T_0 space, Semi-T_1 space, Semi-T_2 space, Semi-T_3 space, Semi-T_4 space, Semi-T_5 space. The mathematician Charles Dorsett[5] introduced the concept of Semi-Regular & Semi-Normal spaces and investigated their properties.

In this paper, analogous to S. N. Maheshwari & R. Prasad's[3] Spaces, we investigate the certain results of these spaces in the topological space.

II. Preliminaries

Throughout this paper (X , τ) is always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X , τ) then C_L(A) & I_N (A) are denote the closure and interior of the set A in the topological space.

2.1 . <u>Semi-T_1 space</u>[28] : A topological space (X , J) is said to be **Semi-T_1 space** if and only if given any pair of distinct points x & y of X, ∃ two Semi-nbd of one containing the point 'x' but not 'y' and the other containing the point 'y' but not 'x'.

Or, A topological space (X , J) is **Semi-T_1 space** if and only if given any pair of distinct points x & y of X, ∃ two Semi-open sets of one containing the point 'x' but not 'y' and the other containing the point 'y' but not 'x'.

Or, A topological space (X , J) is **Semi-T_1 space** if and only if ∃ two Semi-open set G & H such that $x \in G$ but $y \notin G$ and $y \in H$ but $x \notin H$.

2.2 . <u>Proposition</u> : The set of real space R with the usual topology U that is (R , U) is Semi-T_1 space.

<u>Proof</u> : Let x & y be any two distinct real numbers and let $y > x$ and also let $(y - x) = k$, then $G =]\ x - \frac{k}{4}\ ,\ x + \frac{k}{4}\ [$ & $H =]\ y - \frac{k}{4}\ , y + \frac{k}{4}\ [$ are two Semi-open set such that $x \in G$ but $y \notin G$ and $y \in H$ but $x \notin H$.

Hence, the set of real space R with the usual topology U that is (R , U) is Semi-T_1 space.

2.3 . <u>Proposition</u> : A topological space (X , J) is said to be Semi-T_1 space if and only if every singleton subset {x} of the set X is Semi-closed set in J.

Or, all the finite sets are Semi-closed set in Semi-T_1 space.

<u>Proof</u> : Let every singleton subset of the set X be Semi-closed set then we have to show that the space is Semi-T_1 space.

Let x & y be any two distinct points of the set X and since every singleton subset that is {x} of the set X be Semi-closed set. So, (X – {x}) is a Semi-open set which contain the point 'y' but does not contain the point 'x'.

Similarly, since every singleton subset that is {y} of the set X be also Semi-closed set. So, (X – {y}) is a Semi-open set which contain the point ' x' but does not contain point 'y'.

Hence, the space is Semi-T_1 space.

Conversely ,

Let the space is Semi-T_1 space & let 'x' be any point of the set X then we have to show that the singleton set {x} is a Semi-closed set that is (X – {x}) is a Semi-open set.

Let y ∈ (X – {x}) then y ≠ x and since the set X is Semi-T_1 space. So, ∃ a Semi-open set G_y such that the point 'y' ∈ G_y but 'x' ∉ G_y .

It follows that, 'y' ∈ $G_y \subset$ (X – {x}) & (X – {x}) is a Semi-open set.

Hence, the singleton set {x} is a Semi-closed set.

2.4 . <u>Corollary</u> : Since, the union of finite number of Semi-closed set is Semi-closed set. So, a space X is Semi-T_1 space if and only if every finite subset of X is Semi-closed set.

2.5 . <u>Pre-Semi open function</u> : Let (X , J_1) and (Y , J_2) be two topological spaces and consider the function f : (X , J_1) ⟶ (Y , J_2) then the function f is said to be Pre-Semi open function if f maps each Semi-open set of (X , J_1) to a Semi-open set of (Y , J_2).

2.6 . <u>Semi-homeomorphism</u> : Let (X , J_1) and (Y , J_2) be topological spaces and consider the function f : (X , J_1)⟶ (Y , J_2) then the function f is said to be a Semi-homeomorphism of (X , J_1) onto (Y , J_2) if the function f is one-one and onto , irresolute & Pre-Semi open function.

2.7. <u>Semi-homeomorphic</u> : A topological spaces (X , J_1) is Semi-homeomorphic with other topological spaces (Y , J_2) if ∃ a Semi-homeomorphism of (X , J_1) onto (Y , J_2).

Since, each homeomorphism of (X , J_1) onto (Y, J_2) is a Semi-homeomorphism of (X , J_1) onto (Y, J_2) but not conversely.

2.8 . <u>Semi-topological property</u> : A property of a topological space X is said to be Semi-topological property if and only if it is preserved under Semi-homeomorphism

2.9. <u>Proposition</u> : The property of a topological space being a Semi-T_1 space is preserved under one-one & onto Pre-Semi open function. So, it is a Semi-topological property.

Or, the homeomorphic image of Semi-T_1 space is a Semi-T_1 space.

Proof : Let (X , J) be a Semi-T_1 space and let f be a one-one Pre-Semi open function between one topological space (X , J) onto another topological space (Y , J*) then we have to show that (Y , J*) is also a Semi-T_1 space.

Let y_1 & y_2 be any two distinct points of the set Y. Since, the function f is one-one and onto. So, ∃ , distinct points x_1 & x_2 of the set X such that $f(x_1) = y_1$ & $f(x_2) = y_2$.

Since, (X , J) be a Semi-T_1 space. So, ∃ two Semi-open set G & H ∈ J such that $x_1 \in G$ but $x_2 \notin G$ and $x_2 \in H$ but $x_1 \notin H$.

Since, the mapping f is a Pre-Semi open function. So, $f(G)$ & $f(H) \in J^*$ is a Semi-open set s. t. $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$ & $y_2 = f(x_2) \in f(H)$ but $y_1 = f(x_1) \notin f(H)$. **Hence, (Y , J*) is also a Semi-T_1 space.**

Since, the property of being Semi-T_1 space is preserved under one-one & onto Pre-Semi open function. So, it is preserved under Semi-homeomorphism**.**

Hence, it is a Semi-topological property.

2.10 . Proposition : Every metric space is a Semi-T_1 space in Semi-topological spaces.

Proof : Let (X , d) be a metric space and let x be any arbitrary point of the set X then we have to show that {x} is Semi-closed set.

Let 'y' be any point of the set X different from the point 'x' and let d(x , y) = k then the sphere $S(\frac{y}{2}, k)$ is a Semi-open set of the point 'y' which does not contain the point 'x'.

Hence, the point 'y' is not a Semi-limit point of the singleton set {x} so clearly the point 'x' cannot be Semi-limit point of {x}. It follows that, no point of set X can be the Semi-limit point of {x}. So, $SC_L\{x\} = \{x\}$.

Hence, the singleton set {x} is a Semi-closed set.

Therefore, every metric space is a Semi-T_1 space in Semi-topological spaces.

2.11 . Proposition : Every finite Semi-T_1 space is discrete space in topological space.

Proof : Let (X , J) be a Semi-T_1 space and the set X is finite. Since, the space is Semi-T_1 space. So, every singleton subset of the set X is Semi-closed set and consequently every finite subset of the set X is Semi-closed set and the set X is finite. So, every subset of the set X is Semi-closed set in the topological space. **Hence, the space must be discrete.**

Therefore, every finite Semi-T_1 space is discrete space.

2.12 . <u>Proposition</u> : Let J and J^* be two topologies on a nonempty set X and let J^* be finer than J that is $J \subset J^*$ then if J is Semi-T_1 space then J^* is also a Semi-T_1 space.

<u>Proof</u> : Let x & y be any two distinct points of the set X.

Since, (X , J) is Semi-T_1 space. So, $\exists$ two Semi-open set G & H $\in$ J such that x $\in$ G but y $\notin$ G & y $\in$ H but x $\notin$ H and $J \subset J^*$ so G & H are also Semi-open sets s. t. x $\in$ G but y $\notin$ G & y $\in$ H but x $\notin$ H in J^*.

Hence, (X , J^*) is a ST_1-space in topological space.

2.13. <u>Proposition</u> : Every Semi-T_1 space is a Semi-T_0 space in Semi-topological space.

<u>Verification</u> : Let (X , J) be a Semi-T_1 space and let x & y be any two distinct points of the set X. Since, (X , J) is Semi-T_1 space. So, $\exists$ two Semi-open sets G & H $\in$ J s. t x $\in$ G but y $\notin$ G & y $\in$ H but x $\notin$ H.

This implies that, $\exists$ a Semi-open set G $\in$ J s. t x $\in$ G but y $\notin$ G.

So, (X , J) be a Semi-T_0 space in the Semi-topological space.

Hence, every Semi-T_1 space is a Semi-T_0 space in the Semi-topological space.

Remark :-

$$\text{“ } T_2\text{-space} \longrightarrow \text{Semi-}T_2\text{-space}$$
$$\downarrow \qquad\qquad \downarrow$$
$$T_1\text{-space} \longrightarrow \text{Semi-}T_1\text{-space}$$
$$\downarrow \qquad\qquad \downarrow$$
$$T_0\text{-space} \longrightarrow \text{Semi-}T_0\text{-space ”}$$

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